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Summary

Simple rail guns and rail guns with normally conducting and superconducting augmentation have been analyzed using an energy approach. Ideal launch efficiencies, neglecting Joule losses and assuming constant rail current during the launch, are shown to be limited to 50 percent for normally conducting systems, but can approach 100 percent for systems with superconducting augmentation. Application of the technology to pulse power switches, such as the Marshall rail switch, is discussed.

The Augmented Rail Gun

Figure 1 shows schematically the augmented rail gun configuration. The box labeled energy source contains all the components of the system required to provide properly pulsed energy (current) to the slider rails creating the magnetic field which transmits the mechanical energy to the projectile via Lorentz forces. The expression for the Lorentz force in a simple rail gun without augmentation is¹

$$F_{EM} = \frac{1}{2} L' I^2 \quad (1)$$

where L' is the inductance per unit length of the rails and I is the current in the rails.

Augmentation provides an additional field which enhances the energy transfer to a projectile. Since the augmentation field exists both in front of as well as in back of the projectile, it is twice as effective as the rail field itself.¹

Energy Analysis of Ideal Rail Gun Configurations

In this analysis we will assume that the geometries of the configurations are fixed, that is, suitable support of the rails and augmentation coil(s) is provided in the design to prevent mechanical energy conversion into deformation energy in both systems.

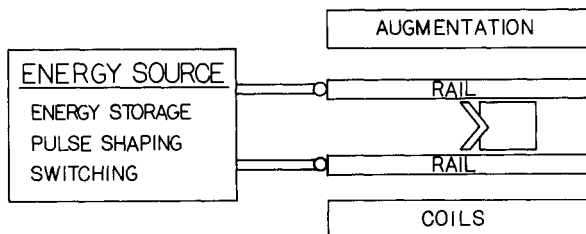


Fig. 1. Schematic layout of the augmented rail gun.

The Joule heating losses will not be analyzed specifically since the design of the rails and the normally conducting augmentation coil(s) strongly affect this loss.

Simple Rail Gun

In this case the energy source delivers a current I to the rails. Assuming a constant current I for the launch, the energy storage in the magnetic field W_m is

$$W_m = \int dW_m = \frac{1}{2} I^2 \int_0^L dL = \frac{1}{2} I^2 L \quad (2)$$

where L is the self inductance of the rail circuit at the end of the launch, i.e., when the projectile leaves the rails. The assumption of constant current during launch simplifies the arguments to be made here, however, an analysis of the system including the driving coil inductance of the source has been made elsewhere.² This analysis shows that the constant current assumption does not significantly affect the conclusions reached in this paper.

The differential magnetic flux generated in this system $d\phi = I dL$ can now be used to calculate the work done by the energy source, W_s , in providing the magnetic field energy and the mechanical work on the projectile. That is

$$W_s = \int dW_s = I \int (d\phi/dt) dt = I \int_0^L I dL = I^2 L \quad (3)$$

The mechanical work W_M provided the projectile (neglecting frictional, etc. losses) is

$$W_M = W_s - W_m = \frac{1}{2} I^2 L \quad (4)$$

It is important to note that the magnetic energy W_m stored in the rail field at the end of the launch is lost by dissipation in the muzzle resistor.

Rail Gun With Normal Conducting Augmentation Coils

In this case, the energy sources must provide a constant current in the rails, I , and a constant current in the augmentation coils, I_A , during the launch.

The change in energy stored in the magnetic fields (neglecting the magnetic energy stored in the self inductance field of the augmentation coils, which remains constant throughout the process and does not contribute to the launch energies in this case) is

$$W_m = \int dW_m = \frac{1}{2} I^2 \int_0^L dL + I I_A \int_0^M dM = \frac{1}{2} I^2 L + I I_A M \quad (5)$$

where L and M are, respectively, the self and the mutual inductance of the circuit at the end of the launch, i.e., $L = \ell L'$ and $M = \ell M'$, ℓ is the length of the rails, and L' and M' are the self and mutual inductance per unit length.

Again the energy sources (rail and augmentation circuit) must provide both the magnetic field energy and the mechanical energy to the projectile. Since the differential magnetic flux through the rail circuit and the augmentation coils in this case is

$$d\phi = I dL + I dM + I_A dM \quad (6)$$

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the work performed by the energy sources can be easily shown to be

$$W_S = I^2 L + 2 I I_A M \quad (7)$$

The mechanical work W_M is as before,

$$W_M = W_S - W_m = \frac{1}{2} I^2 L + I I_A M \quad (8)$$

As in the case of the simple rail gun, the mechanical work equals the energy stored in the magnetic field. The latter is again dissipated by Joule heating after the projectile leaves the rails and is therefore lost. The advantage of normal conducting augmentation coils appears to be that a greater energy can be imparted to the projectile at the same current output from the rail energy source at the expense of an increased total power consumption. Of course, Joule heating losses will be greater in the system since magnetic fields cannot be maintained without losses in the normal conductors used for augmentation.

Rail Gun With Superconducting Augmentation Coils

Since a superconducting coil will retain its stored energy without loss, the self inductive magnetic energy may be extracted from the energy source before a projectile is launched. As we will show subsequently, this stored energy remains in the coil after the projectile is launched, except for small losses due to fluxon motion, as long as the coil is maintained in the superconducting state.

Initially, therefore, a stored energy in the self inductance field of the superconducting coil exists equal to $(1/2) I_{SO}^2 L_S$, where I_{SO} is the superconducting current and L_S is the self inductance of the superconducting coil.

As the projectile is launched by a constant current through the rails, I , the change in magnetic field energy is

$$dW_m = \frac{1}{2} I^2 dL + I I_S dM + I M dI_S + L_S I_S dI_S \quad (9)$$

where we note the current in the superconducting coil I_S varies as a function of I , L_S , and M , with $M = M'x$ where x is the displacement along the rail.

To evaluate this variation in supercurrent, we invoke the unique superconducting property of zero resistance. That is, using Faraday's law, there can be no induced emf in a superconducting coil.

$$\oint \mathbf{E} \cdot d\mathbf{l} = - d\phi / dt = 0 \text{ (superconducting coil)} \quad (10)$$

The flux threading the superconducting coil is constant and in differential form is

$$d\phi = L_S dI_S + I dM = 0 \quad (11)$$

assuming a constant current from the energy source during launch and constant self inductance L_S of the superconducting coil.

Substituting Eq. (11) in Eq. (9) and integrating, we obtain for the energy of the magnetic field W_m (omitting the constant of integration $(1/2) L_S I_{SO}^2$ here and in the work required by the energy source since it is recovered).

$$W_m = \frac{1}{2} I^2 \int_0^L dL - (I^2 \int_0^M M dM) / L_S = \frac{1}{2} I^2 L - \frac{1}{2} M^2 I^2 / L_S \quad (12)$$

The first term of Eq. (12) is identical to Eq. (2) and represents the magnetic energy stored in the field of a simple rail gun (SRG). The second term corresponds to a decrease in the magnetic energy of the superconducting coil.

The differential work required of the energy source dW_S using Eq. (11) is

$$dW_S = I^2 dL + I_S I dM - M I^2 dM / L_S \quad (13)$$

In order to integrate Eq. (13), we obtain an expression for the superconducting current, I_S , as a function of I , L_S , and M by integrating Eq. (11) for constant current I , viz.

$$I_S (M) = I_{SO} - I M / L_S \quad (14)$$

Here I_{SO} is the initial superconducting current and M the mutual inductance during the launch, i.e., $M = M'x$ where x is the displacement along the rail. Using Eqs. (11) and (14) with Eq. (13), we obtain at the end of the launch ($M = M'x$) for the energy required from the energy source,

$$W_S = L I^2 + M I I_{SO} - M^2 I^2 / L_S \quad (15)$$

The mechanical work imparted to the projectile W_M is

$$W_M = W_S - W_m = \frac{1}{2} L I^2 + M I I_{SO} - \frac{1}{2} M^2 I^2 / L_S \quad (16)$$

This calculation has been carried out to the point at which the projectile leaves the rails, but it is important to note that the superconducting coil will recover energy from the magnetic field of the SRG to return to its prelaunch condition. For that reason we have not included the constant term $(1/2) L_S I_{SO}^2$ in Eqs. (12) and (15). The recovered energy E_R is just the second term in Eq. (12), neglecting fluxon motion losses, i.e.,

$$E_R = \frac{1}{2} M^2 I^2 / L_S \quad (17)$$

Thus the energy lost in the muzzle resistor is substantially reduced. An evaluation of this energy saving will be made in a later section.

The superconducting augmented rail gun system, in addition to providing significant advantages in launch efficiencies which will be discussed in the next section, provides the benefits of charging before launching, retention of its stored energy between launches, and lessening the severity of field collapse after launch by recovery of a portion of the collapsing magnetic field energy of the rails.

Comparison of Various Ideal Rail Gun Configurations

For this comparison, we will assume that the currents in the augmentation coil and the rail current are equal.

$$I = I_A \text{ (normal augmentation)} \quad (18a)$$

$$I = I_{SO} \text{ (superconducting augmentation)} \quad (18b)$$

Furthermore, we will use the expression,

$$M = k(LL_A)^{1/2}, \quad (k < 1) \quad (19a)$$

$$M = k(LL_S)^{1/2}, \quad (k < 1) \quad (19b)$$

where k is the coefficient of magnetic coupling, in evaluating the systems.

The source must provide (Eqs. (2) through (4))

$$W_s = W_m + W_M = \frac{1}{2} I^2 L + \frac{1}{2} I^2 L = I^2 L \quad (20)$$

Whereas the energy lost in the resistor is the total magnetic energy

$$W_m = \frac{1}{2} I^2 L \quad (21)$$

Thus, the launch efficiency, defined as the ratio of the mechanical work to the source work, of the ideal SRG is 50 percent. Actual efficiencies will be somewhat less due to resistive heating (Joule) of the rails.

Rail Gun With Normal Conducting Augmentation

The sources must provide, using Eqs. (7), (18a), and (19a)

$$W_s = I^2 L (1 + 2k\sqrt{L_A/L}) \quad (22)$$

and the mechanical energy is from Eq. (8)

$$W_M = \frac{1}{2} I^2 L (1 + 2k\sqrt{L_A/L}) \quad (23)$$

An equal amount of energy is stored in the magnetic field and lost at the end of the launch.

Thus, the launch efficiency of the ideal rail gun with normal conducting augmentation is also 50 percent. Joule losses in the rail circuit are reduced as compared to the SRG because the same launch energy can be achieved with lower currents. However, this gain is offset by Joule heating losses in the augmentation coils. Furthermore, in a non-ideal system the magnetic field energy $(1/2)L_A I_A^2$ stored in the augmentation coil will be lost after each launch and must be resupplied from the power source for the next launch.

Rail Gun With Superconducting Augmentation Coils

The source provides (during each launch and after charging the superconducting coil with $(1/2)L_s I_{s0}^2$ self inductance energy which is recovered) from Eqs. (15), (18b), and (19b)

$$W_s = LI^2(1 - k^2 + k\sqrt{L_s/L}) \quad (24)$$

The mechanical energy is using Eqs. (16), (18b), and (19b)

$$W_M = \frac{1}{2} LI^2(1 - k^2 + 2k\sqrt{L_s/L}) \quad (25)$$

And the magnetic field energy lost at the end of launch is (Eqs. (12) and (19b))

$$W_m = \frac{1}{2} LI^2(1 - k^2) \quad (26)$$

Assuming for purposes of discussion that $k = 0.5$ and $L_s = L$, the launch efficiency of the ideal rail gun with superconducting augmentation coils becomes 70 percent. It should be noted that the values assumed for k and L_s above represent conservative estimates of what can be achieved in practice.

An alternate design would be to place a comparatively large superconducting inductance in series with the augmentation coil in a configuration such that this "ballast" inductor does not see the strong magnetic field of the rails. In this case we have the condition

where L_s^* is now the total inductance of the augmentation circuit, coil plus ballast, and L and M are defined as before. This situation differs from the previous case in that there is weak total flux linkage between the rail and the augmentation circuit. Using Eq. (19b) and neglecting terms of the order M/L_s^* , Eqs. (24) through (26) are replaced, respectively, by

$$W_s = LI^2(1 + M/L) \quad (28)$$

$$W_M = \frac{1}{2} LI^2(1 + 2M/L) \quad (29)$$

$$W_m = \frac{1}{2} LI^2 \quad (30)$$

Assuming $M = L$, a condition which should be realizable without much difficulty with a pair of augmentation coils, the launch efficiency of the ideal rail gun with weakly flux linked superconducting augmentation is 75 percent. As can be seen by comparison with the above calculation, the launch efficiencies of the two superconducting designs are comparable.

However, in the first approach the acceleration of the projectile is largest at the beginning of the launch, the second approach provides for constant acceleration (for constant rail current) along the entire length of the rail, but at the expense of needing an additional coil. It would be interesting to explore the possible effects of linking this coil to the flux of the energy storage coil.

In selecting a final design for a superconducting augmentation system, the effects of "training"³ of the superconducting coil(s) in the time dependent rail field will have to be carefully evaluated. In this preliminary study we have not considered the effects of training although available technology in design and materials will allow the virtual elimination of deleterious training.³

In selecting materials which minimize training effects, the commercially available Nb₃Sn superconductors show virtually no training effects.³ However, Nb₃Sn is extremely brittle and may require substantial support structures to withstand the effects of high g loading in an augmented rail gun system. We note that the new Pd_xCu_{1-x}H technology developed jointly at Benet Weapons Laboratory and the State University of New York at Albany may be useful in rail gun technology.⁴ The superconducting transition temperatures and critical current characteristics of the new Pd_xCu_{1-x}H superconductors are comparable with Nb₃Sn. In addition, the new materials are ductile.

Variable Current Launchers

In practice, actual launch systems operate under conditions of variable currents, although to maximize launch velocities for a given rail length, designers would opt to keep the rail current as high and as constant as possible during the launch.

We have analyzed the case for variable currents in a simple rail gun and a superconducting augmented rail gun elsewhere,² but report here the salient results.

Simple Rail Gun

The system configuration analyzed includes a charged inductor coil, normally contained in the energy source for pulse shaping purposes, feeding the rails. Rail and inductor resistances are neglected.

In this analysis, the current I varies and we introduce the driving coil inductance L_0 in series with the rail inductance L .

The total magnetic energy of the coil-rail system as a function of projectile position is found to be

$$W_{mT} = \frac{1}{2} L_0 I^2 + \frac{1}{2} x L' I^2 \quad (31)$$

where x is the displacement of the slider along the rails, L' is the rail inductance per unit length and the current I is given by

$$I = \frac{I_0}{1 + x(L'/L_0)} \quad (32)$$

where I_0 is the initial current at the beginning of the launch. Since we are dealing with a conservative system, the force on the projectile may be obtained by taking the negative derivative of Eq. (31) with respect to x . The resulting force on the projectile agrees with the force determined from the derivative of the kinetic energy with respect to x from Eq. (4) even though Eq. (4) was derived for the condition of constant current in the rails.

Superconducting Augmented Rail Gun

Including superconducting augmentation in the system, one obtains for the force on the projectile using the same symbols as defined previously

$$F_x = - \frac{dW_{mT}}{dx} = \frac{1}{2} L' I^2 + I I_{s0} M' - x I^2 \frac{M'^2}{L_s} \quad (33)$$

Again Eq. (33), which was obtained by the negative derivative of the total magnetic energy, is identical to the result one obtains by taking the derivative of the kinetic energy in Eq. (16) with respect to x .

The forces derived from the expressions assuming constant current agree with those derived for variable currents, provided the instantaneous currents are used in the latter expressions.

The energy expressions derived assuming constant currents represent integrals over the force, thus they must be modified where the situation of variable currents pertains. However, it is worth noting that the conclusions drawn regarding efficiencies in the case of constant currents remain relatively unaffected by current variability.

Application of Superconducting Augmentation to Switches

The rail switch is used extensively in pulse power applications to transfer energy from the source to the pulsed device.

The arc currents experienced in their use result from two causes, viz. the voltage applied to the switch from the power source and from the collapse of the rail switch's magnetic field. The latter cause is more important in a properly designed rail switch.

We have shown that the magnetic energy lost in such a switch is equal to the kinetic energy of the switch slider at opening. Since the kinetic energy of the slider should be minimized in a properly designed switch by reducing its mass while adjusting its velocity to prevent the establishment of a long lived rail arc, the actual energy savings by themselves may not warrant adding superconducting augmentation.

The main utility of a rail switch with superconducting augmentation is in arc suppression which is accomplished by the minimization of the magnetic energy stored in the switch's rail field. Reduction of the principal cause of arcing should allow lower opening velocities to be used.

It would be interesting to consider the application of this technology to a rotary switch designed for multiple cycling in a pulse power system or to other configurations in which superconducting coils could act as a "sponge" to avoid deleterious flux collapse in pulse power applications.

Design of a Demonstrator Superconducting System

In order to evaluate other technical factors in addition to the substantial enhancement of performance described above, a design of a small superconducting augmented demonstrator was considered.

A simple rail gun of 1 m in length can accelerate a 10 g projectile to about 10^3 m/sec with a rail current of about 1.2×10^5 A. The efficiency of such a system including Joule losses was estimated to be about 30 percent.

The rail bursting force was calculated to be approximately 6×10^4 nt/m. Adding a pair of superconducting augmentation coils, each carrying 5.5×10^3 amperes with 20 turns, placed symmetrically above and below the rails yielded $k = 0.735$ and increased the system efficiency to over 80 percent. Thus for the same rail current, we obtained an augmented projectile velocity of about 1.6×10^3 m/sec. This configuration of coils should not subject the coils to magnetic fields due to rail currents which exceed the critical fields of commercially available superconducting materials, such as NbTi.

An examination of the stresses involved in such an augmented system showed that the coil bursting force was approximately equal to the rail bursting force calculated above. The attractive forces between the rail and the two symmetrically placed superconducting coils balance. This analysis indicated that a lightweight support structure could be utilized in the augmentation system. A preliminary design, incorporating this support into the Dewar system, led to an estimate of cryogenic requirements well within the capacity of the relatively small LHe refrigerator available in our laboratory.

Thus this relatively crude evaluation of an actual system suggests that the substantial performance enhancements predicted by our analysis of superconducting augmentation can be achieved in an actual design. Experimental studies are planned to evaluate the feasibility and to technical factors of such a design.

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